Optimally Robust Estimation by means of R Packages

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Outline

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Infinitesimal Robust Setup

- Smoothly parameterized family of probability measures

$$\mathcal{P} = \{P_\theta \,|\, \theta \in \Theta\} \qquad \Theta \subset \mathbb{R}^k \text{ (open)}$$

  with L_2 derivative $\Lambda_\theta \in L_2^k(P_\theta)$, $\mathrm{E}_\theta \Lambda_\theta = 0$ and

- Fisher information of full rank

$$\mathcal{I}_\theta = \mathrm{E}_\theta \Lambda_\theta \Lambda_\theta^\tau \qquad \mathcal{I}_\theta \succ 0$$

- Infinitesimal gross error model with radius $\mathrm{r} \in (0, \infty)$

$$\mathrm{U}(\theta, \mathrm{r}) = \left\{ (1 - \mathrm{r}/\sqrt{\mathrm{n}})_+ \mathrm{P}_\theta + (1 \wedge \mathrm{r}/\sqrt{\mathrm{n}})\, \mathrm{Q} \,\middle|\, \mathrm{Q} \in \mathcal{M}_1(\mathcal{A}) \right\}$$

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Definition

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Asymptotically linear (AL) estimator:

$$\sqrt{n}\,(S_n - \theta) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \psi_\theta(y_i) + o_{P_\theta^n}(n^0)$$

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Optimally Robust Estimators

- Optimally robust ICs $\tilde{\eta}_\theta$ w.r.t. maximum asymptotic MSE for various models (including regression and time series models) are derived in Rieder (1994), Rieder (2003) and Kohl (2005)

- Estimator construction by means of the one-step method

$$S_n(y_1, \ldots, y_n) = \hat{\theta}_n(y_1, \ldots, y_n) + \frac{1}{n}\sum_{i=1}^{n} \tilde{\eta}_\theta(y_i)$$

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R Packages

ROptEst: Optimally robust estimation using S4 classes and methods

RobLox: Optimally robust ICs for normal location and scale

ROptRegTS: Optimally robust estimation for regression-type models using S4 classes and methods

RobRex: Optimally robust ICs for normal linear regression with unknown error scale

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